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PROGRAMME DE RECHERCHE
NUMÉRIQUE POUR L'EXASCALE

# Multi-fidelity Variational Data Assimilation for Ocean forecasting

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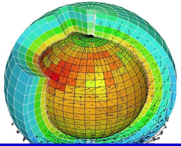
# Data assimilation



**Observations**

- Sparse
- Limited spatial coverage
- Limited resolution

**Background**

- Assumptions
- Limited information on model inputs

Statistical aggregation

**Analysis**

- Uncertainty quantification *(error model)*
- Inverse modeling *(estimation procedure)*

[Sources: Illustration by Laurent Fairhead, Laboratoire de Météorologie Dynamique, Paris; MEOP/CHRISTOPHE GUINET; https://www.encyclopedie-environnement.org/air/observations-meteo-au-sol/; CNES, Thierry Lafon ]

# Variational Data Assimilation - 4DVar

$$J(\mathrm{x}_0) = \frac{1}{2}(\mathrm{x}_0 - \mathrm{x}^b)^T B^{-1}(\mathrm{x}_0 - \mathrm{x}^b) + \frac{1}{2}\sum_{n=1}^{N}(H_n M_{0,n}(\mathrm{x}_0) - \mathrm{y}_n)^T R_n^{-1}(H_n M_{0,n}(\mathrm{x}_0) - \mathrm{y}_n) \quad (1)$$

- Problem with size $10^6 - 10^9$
- Model $H_n M_{0,n}$ *weakly non-linear*
- Fewer observations in terms of the degrees of freedom
- $J$ not necessarily *quadratic* $\Rightarrow$ Gauss-Newton method with successive quadratic approximations
- Access to the gradient of $J$

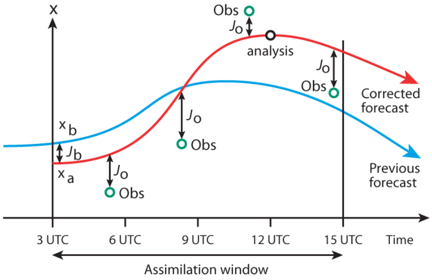
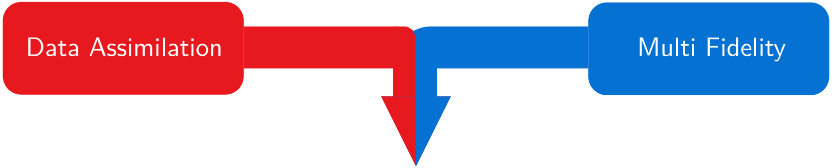



Figure 1: Illustration of the 4DVar principle

[Source: ECMWF, 2017, https://www.ecmwf.int/en/about/media-centre/news/2017/20-years-4d-var-better-forecasts-through-better-use-observations]

# Using multifidelity in a Variational DA framework




## Why ?

### Main objectives

1. Decrease the computational cost of the product $H_n M_{0,n}$ by using substitutes with different fidelities
2. Choosing fidelities depending on a error budget
3. Developping a proof of convergence for the multifidelity incremental 4D-Var

# Current work

**Inexact Conjugate Gradient** focusing on the inaccuracy budget

Serge Gratton, Ehouarn Simon, David Titley-Peloquin et Philippe L. Toint. « Minimizing convex quadratics with variable precision conjugate gradients ». In : Numerical Linear Algebra with Applications 28.1 (2021)

**Local multifidelity correction** using trust-region optimisation

Christopher C. Fischer, Ramana V. Grandhi et Philip S. Beran. « Bayesian Low-Fidelity Correction Approach to Multi-Fidelity Aerospace Design ». In : 58th AIAA/ASCE/AHS/ASCStructures, Structural Dynamics, and Materials Conference.