

# High order discretization

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# Work during the PhD thesis

*Mathematical modeling, simulation and order reduction of ocular flow and their interactions: building the digital twin of the eye*

- **Modeling and simulation** of heat transfer coupled with aqueous humor flow in the **anterior chamber**.
- Employ adapted **discretization and resolution techniques** to compute numerical solutions (*WP1, WP3*).
- **Verification and validation** of the numerical models developed.
- **Model order reduction** with the Certified Reduced Basis Method (*WP2*).
- **Sensitivity analysis** (*WP2, WP6*) to assess impact of parameters on quantities of interest, and enhance development of clinical treatments.
- Contributed benchmarks to **deliverable D7.1** (*WP1, WP2*).

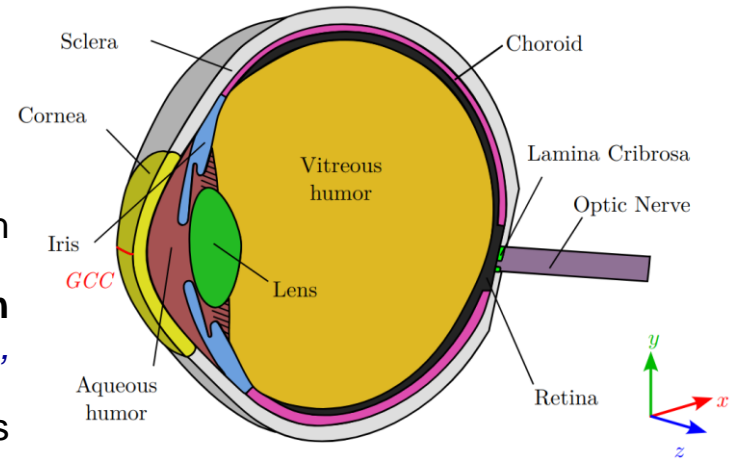


Fig. 1: Vertical cut of the geometrical model of the human eye, with the different regions of the eye.

$$\rho(\mathbf{u} \cdot \nabla)\mathbf{u} - \nabla \cdot (2\mu \underline{\underline{D}}(\mathbf{u}) - p\mathbf{I}) = -\rho\beta(T - T_{ref})\mathbf{g} \quad \text{in } \Omega_{AH},$$

$$\nabla \cdot \mathbf{u} = 0 \quad \text{in } \Omega_{AH},$$

$$\rho C_p \mathbf{u} \cdot \nabla T - k \nabla^2 T = 0 \quad \text{in } \Omega.$$

# Scalability analysis with Feel++ [github.com/feelpp/feelpp](https://github.com/feelpp/feelpp)

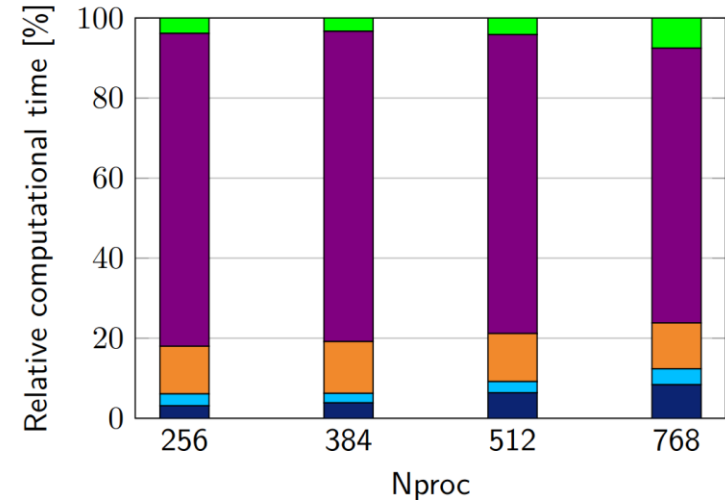
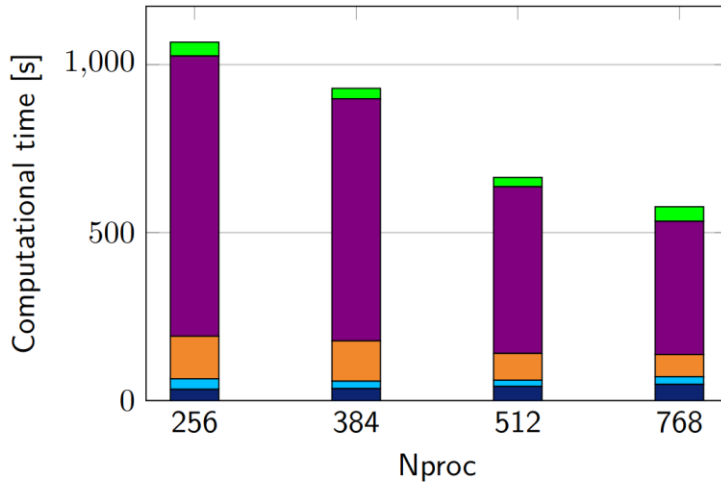


Fig. 2: Absolute (left) and relative (right) computational time for the coupled heat-fluid model in the standing position (Mesh with  $1.51 \times 10^8$  elements, and  $P_1$ - $P_2$ - $P_1$  discretization).

■ Mesh loading 
 ■ Data Structures 
 ■ System Assembly 
 ■ Solve 
 ■ Post Process

# Upcoming work

- Research Engineer (*WP7* and *WP1*)
  - High order method working with Exa-DI working group on HOM
    - Working with Kokkos and have portable performance on CPUs and GPUs
    - Studying work done in the context of ECP (libceed, mfem,...)
    - Contributing to corresponding SDK
  - Packaging
    - Spack, guix-hpc, containers (apptainer)
    - Deploying on supercomputer
  - Benchmarking (D7.1) activities
    - Benchmarking, Profiling
    - Automating benchmarking and associated reporting (e.g. tool developed at Unistra)



PROGRAMME  
DE RECHERCHE  
NUMÉRIQUE  
POUR L'EXASCALE

Thanks for your attention !